

COMP219: Artificial Intelligence

Lecture 13: Game Playing

Overview

- Last time
 - Search with partial/no observations
 - Belief states
 - Incremental belief state search
 - Determinism vs non-determinism
- Today
 - We will look at how search can be applied to playing games
 - Types of games
 - Perfect play
 - minimax decisions
 - alpha-beta pruning
 - Playing with limited recourses

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Games and Search

- In search we make **all** the moves. In games we play against an “**unpredictable**” opponent
 - Solution is a **strategy** specifying a move for **every possible** opponent reply
- Assume that the opponent is intelligent: **always** makes the **best** move
- Some method is needed for selecting **good** moves that stand a good chance of achieving a winning position, **whatever** the opponent does!
- There are time limits, so we are unlikely to find goal, and must approximate using **heuristics**

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Types of Game



- In some games we have perfect information – the position is known completely
- In others we have imperfect information: e.g. we cannot see the opponent’s cards
- Some games are deterministic – no random element
- Others have elements of chance (dice, cards)

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Types of Games

	deterministic	chance
perfect information	chess, checkers, go, othello	backgammon monopoly
imperfect information	battleships, blind tictactoe	bridge, poker, scrabble nuclear war

We will consider:

- Games that are:
 - Deterministic
 - Two-player
 - Zero-sum
 - the utility values at the end are equal and opposite
 - example: one wins (+1) the other loses (-1)
 - Perfect information
- E.g. Othello, Blitz Chess

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Problem Formulation

- Initial state
 - Initial board position, player to move
- Transition model
 - List of (move, state) pairs, one per legal move
- Terminal test
 - Determines when the game is over
- Utility function
 - Numeric value for terminal states
 - e.g. Chess +1, -1, 0
 - e.g. Backgammon +192 to -192

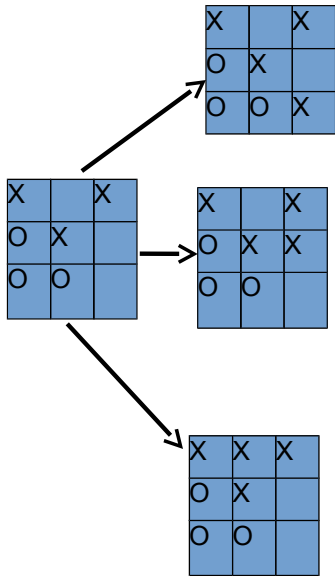
Noughts and Crosses

X		X
O	X	
O	O	

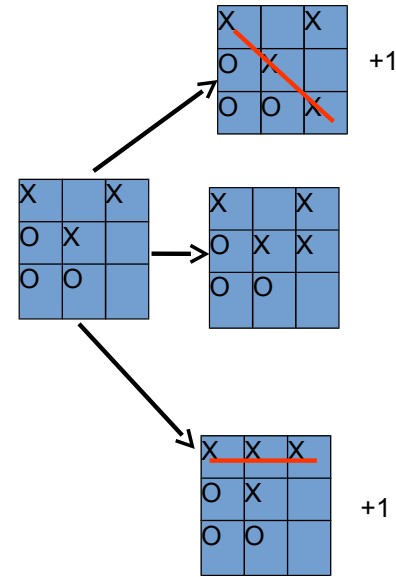
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Noughts and Crosses



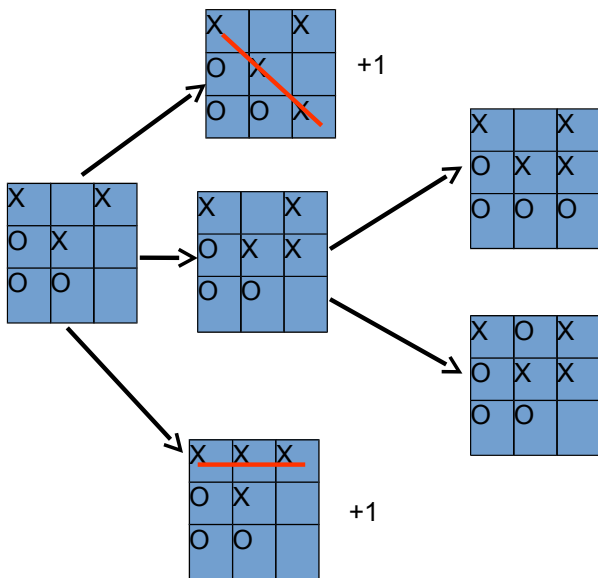
Noughts and Crosses



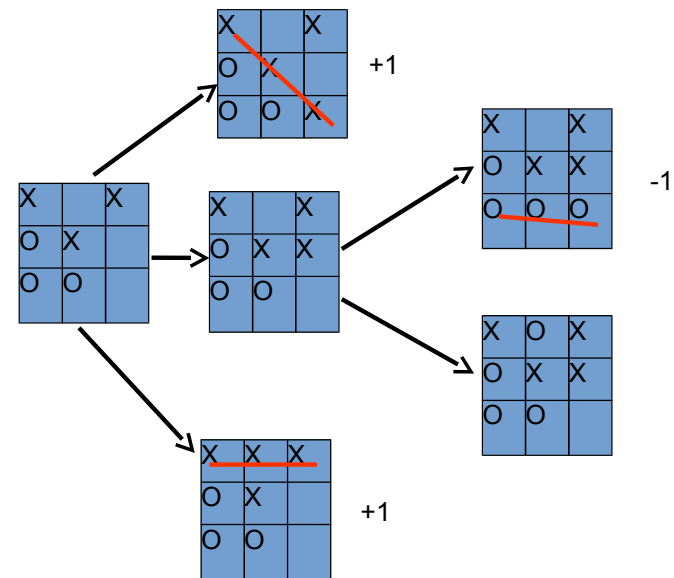
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Noughts and Crosses



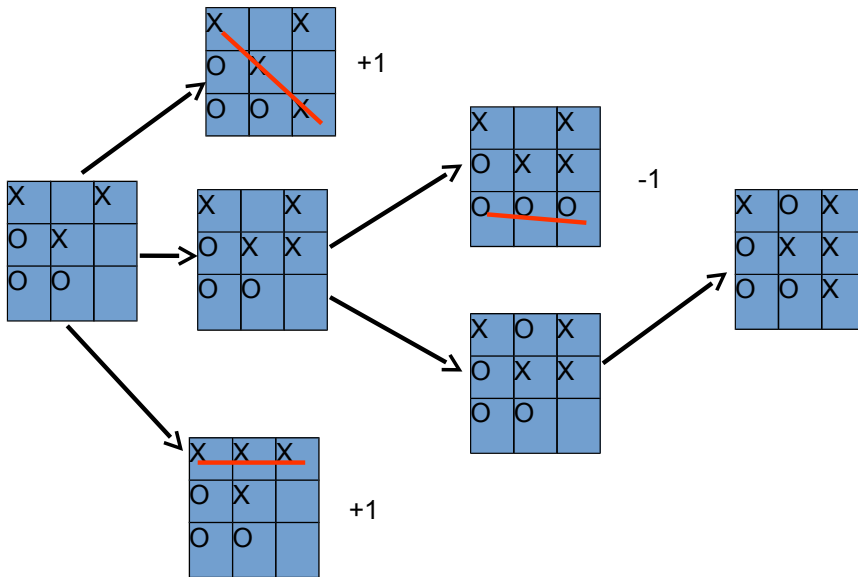
Noughts and Crosses



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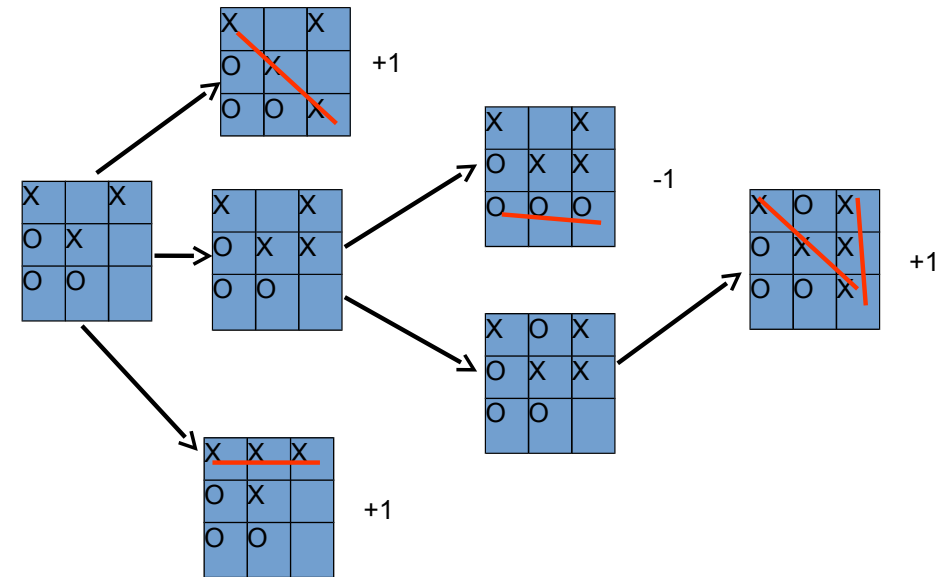
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Noughts and Crosses



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Noughts and Crosses



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Game Tree

- Each level labelled with **player to move**
- Each level represents a **ply**
 - Half a turn
- Represents what happens with **competing agents**

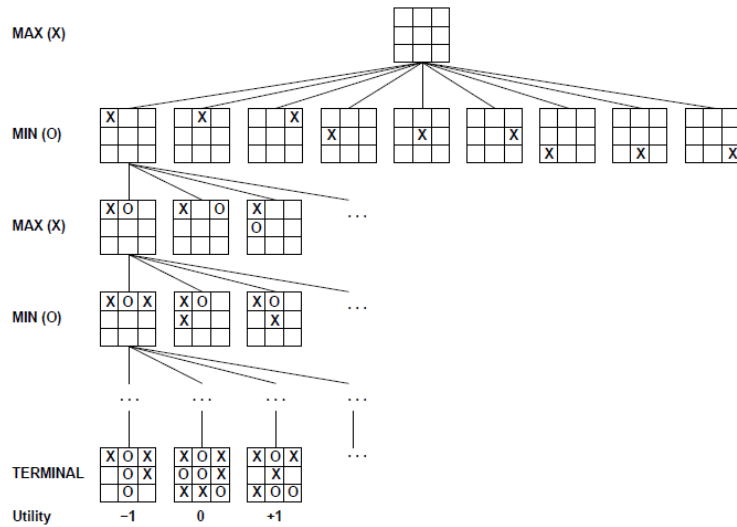
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Introducing MIN and MAX

- MIN and MAX are two players:
 - MAX wants to **win** (maximise utility)
 - MIN wants **MAX to lose** (minimise utility for MAX)
 - MIN is the Opponent
- Both players will play to the best of their ability
 - MAX wants a strategy for maximising utility assuming MIN will do best to minimise MAX's utility
 - Consider **minimax** value of each node

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Example Game Tree



Minimax Value

- Utility for MAX of being in that state assuming both players play optimally to the end of the game
- Formally:

$$\text{MinimaxValue}(n) = \begin{cases} \text{Utility}(n) & \text{Terminal} \\ \max_{s \in \text{Successors}(n)} \text{MinimaxValue}(s) & \text{MAX} \\ \min_{s \in \text{Successors}(n)} \text{MinimaxValue}(s) & \text{MIN} \end{cases}$$

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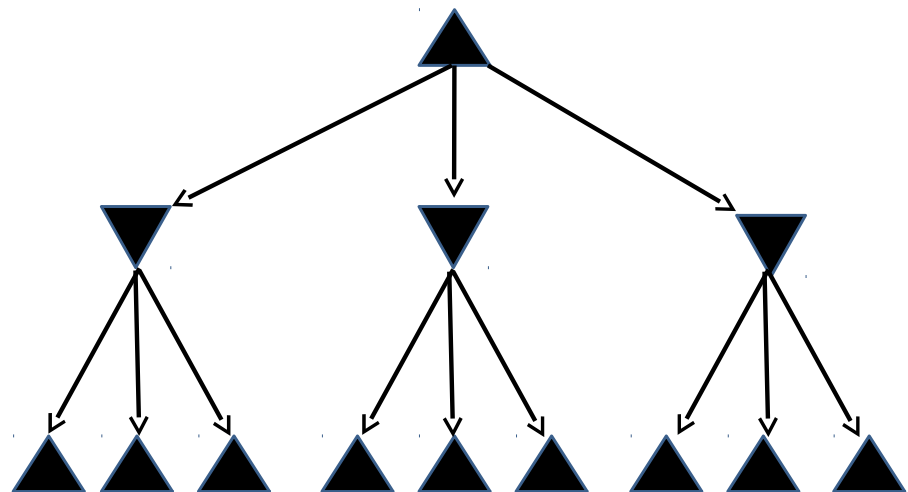
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Minimax Algorithm

- Calculate minimaxValue of each node recursively
- Depth-first exploration of tree
- Game tree as *minimax tree*
- *Max Node*:
- *Min Node*



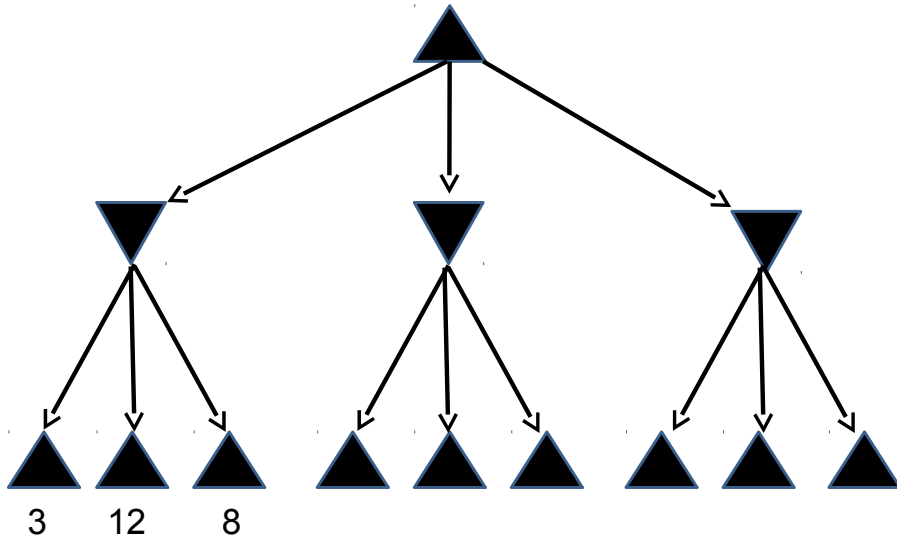
Minimax Tree



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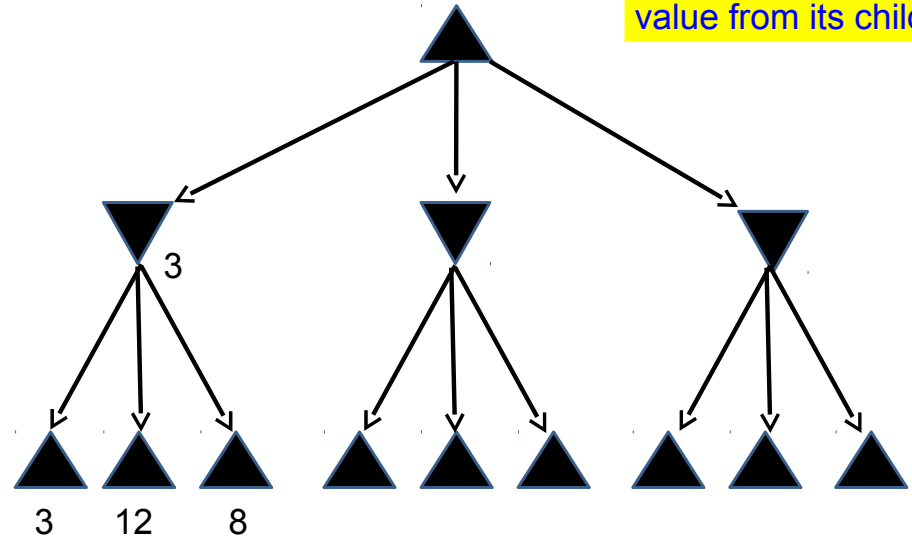
Minimax Tree



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Minimax Tree

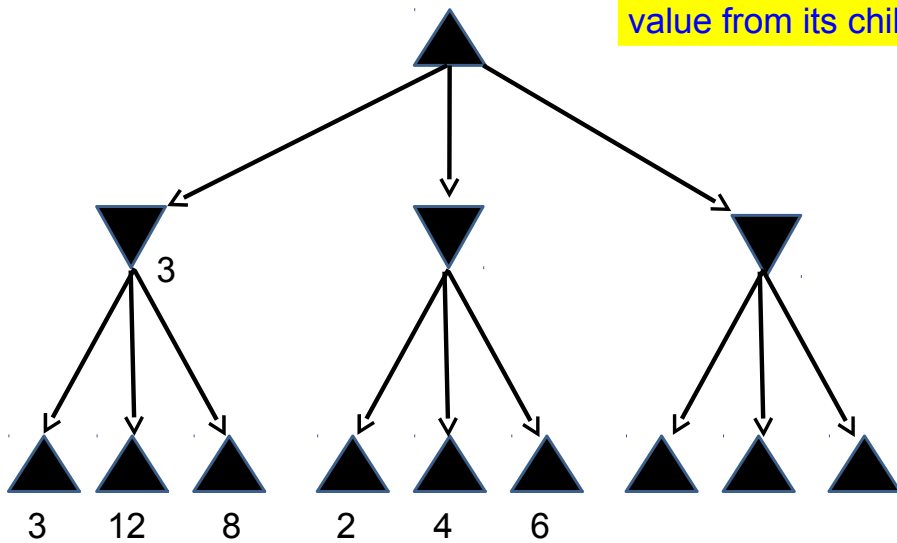
Min takes the **lowest** value from its children



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Minimax Tree

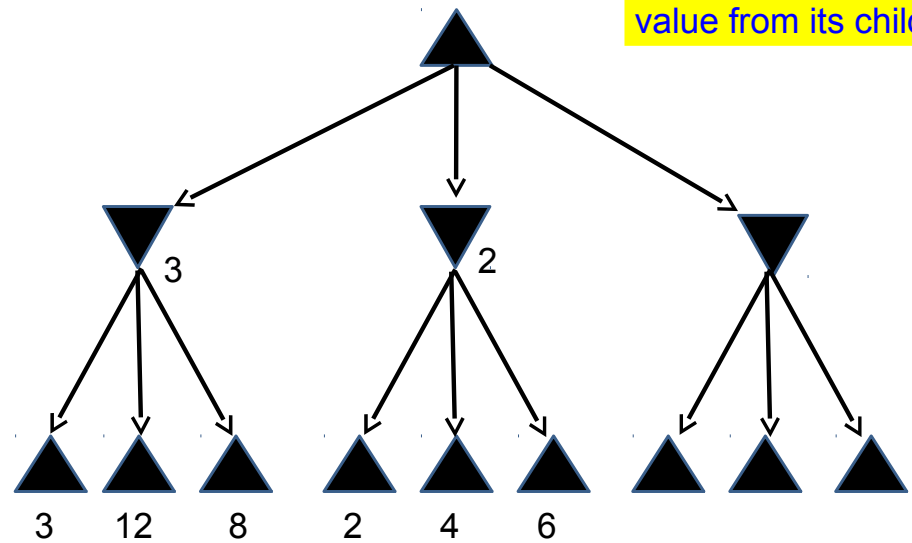
Min takes the **lowest** value from its children



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Minimax Tree

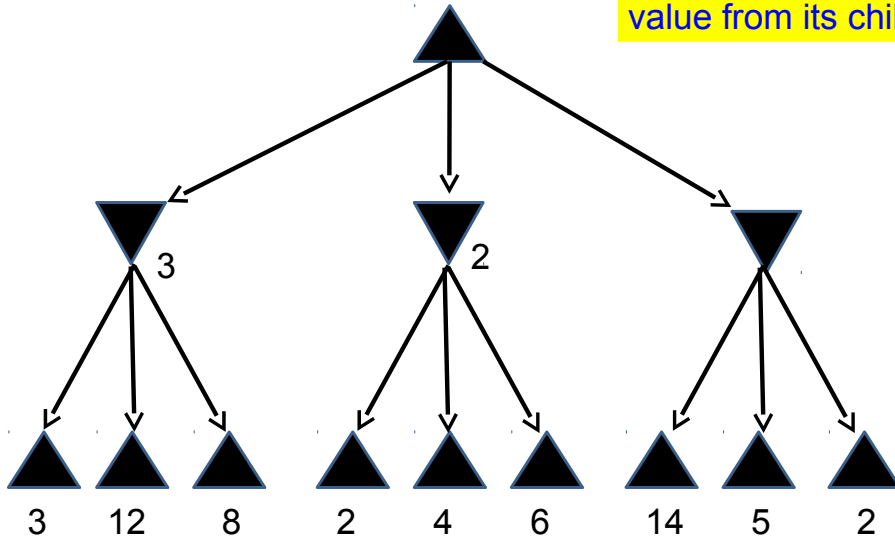
Min takes the **lowest** value from its children



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Minimax Tree

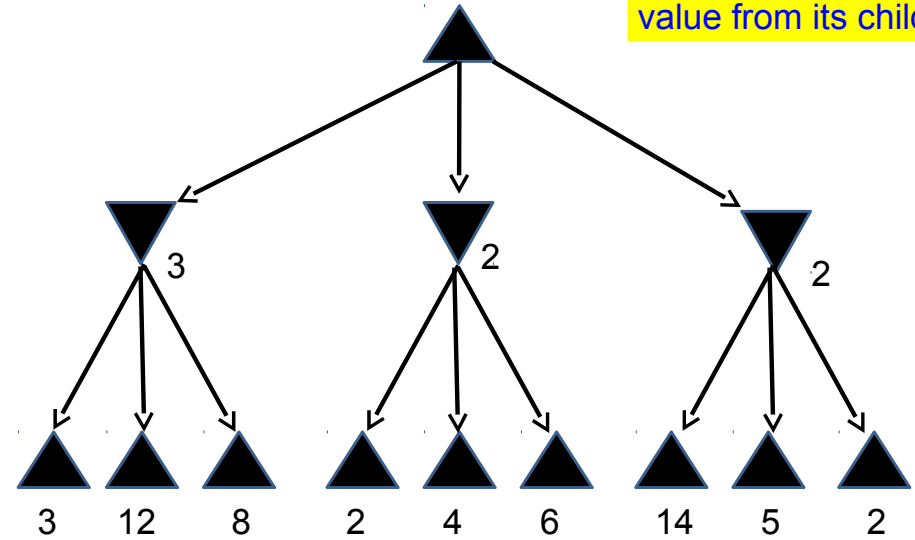
Min takes the **lowest** value from its children



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Minimax Tree

Min takes the **lowest** value from its children

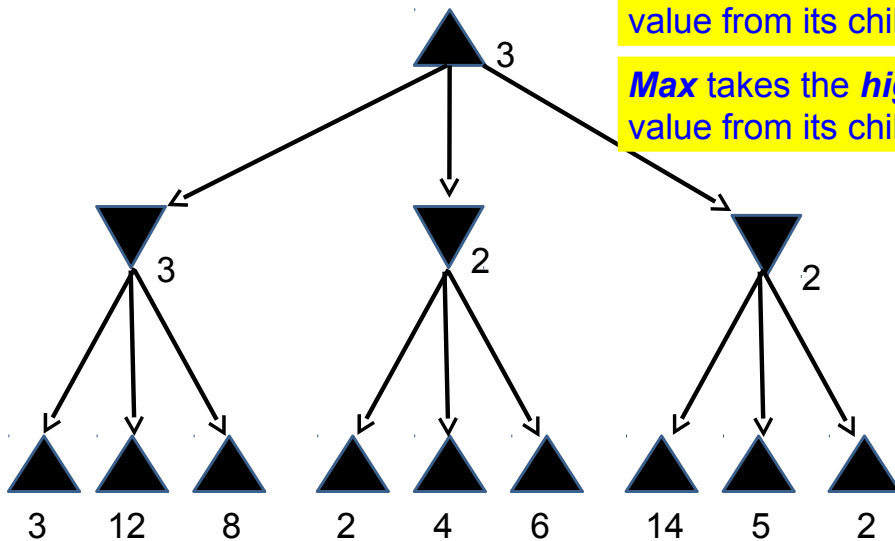


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Minimax Tree

Min takes the **lowest** value from its children

Max takes the **highest** value from its children



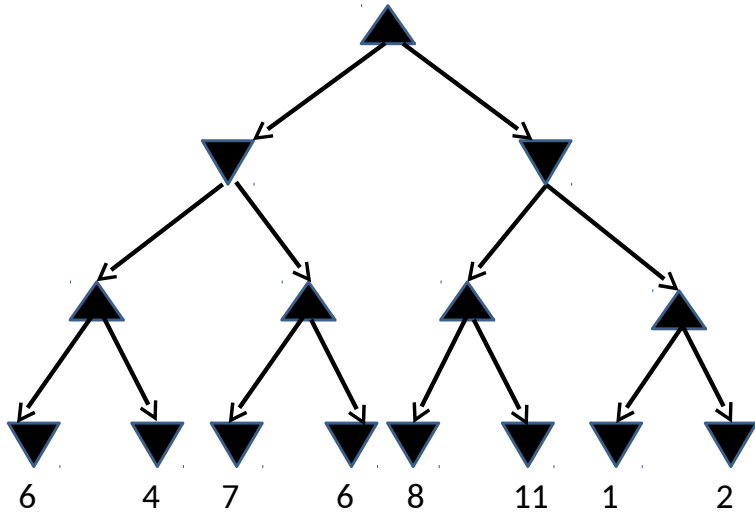
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Exercise

- Perform the minimax search algorithm on the following tree to get the minimax value of the root:

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Exercise



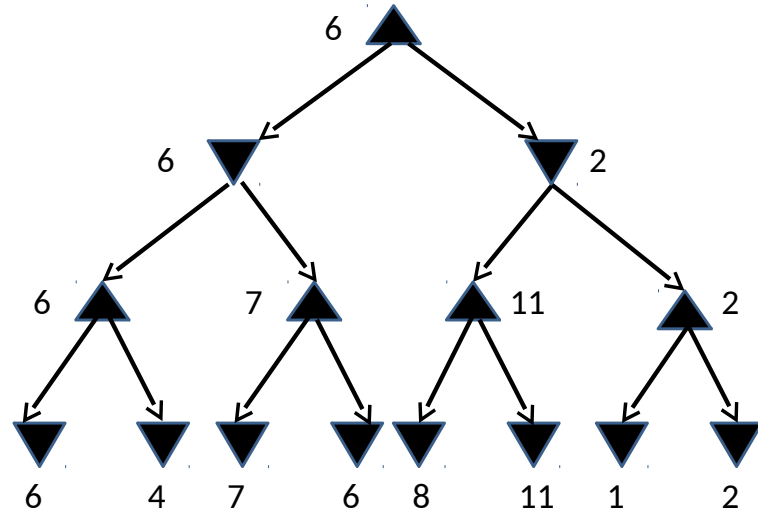
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Properties of Minimax

- Complete, if tree is **finite**
- Optimal, **against an optimal opponent**. Otherwise??
 - No. e.g. expected utility against **random** player
- Time complexity: b^m
- Space complexity: bm (depth-first exploration)
 - For chess, $b \approx 35$, $m \approx 100$ for “reasonable” games
 - **Infeasible** – so typically set a limit on look ahead. Can still use minimax, but the terminal node is **deeper** on every move, so there can be surprises. **No longer optimal**
- But do we need to explore **every** path?

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Exercise



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Pruning



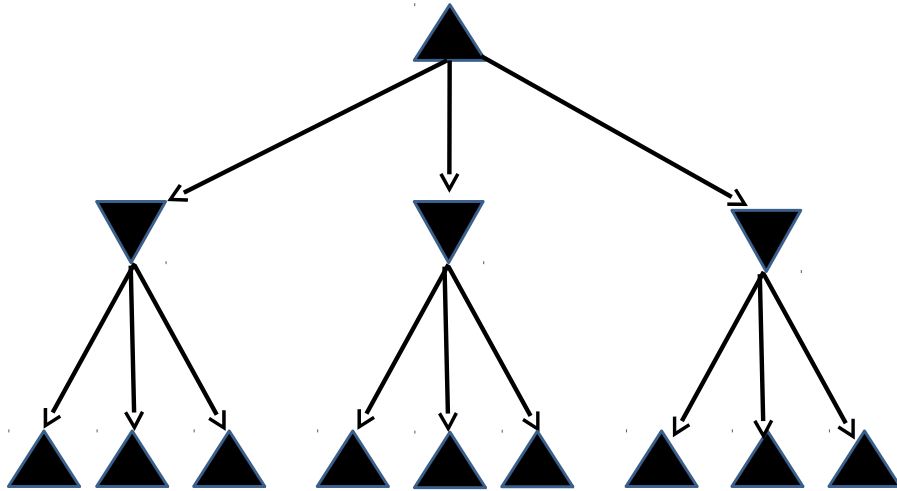
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- Basic idea:
 - If you know half-way through a calculation that it will succeed or fail, then there is no point in doing the rest of it
- For example, in Java it is clear that when evaluating statements like


```
if ((A > 4) || (B < 0))
```
- If A is 5 we do not have to check on B!

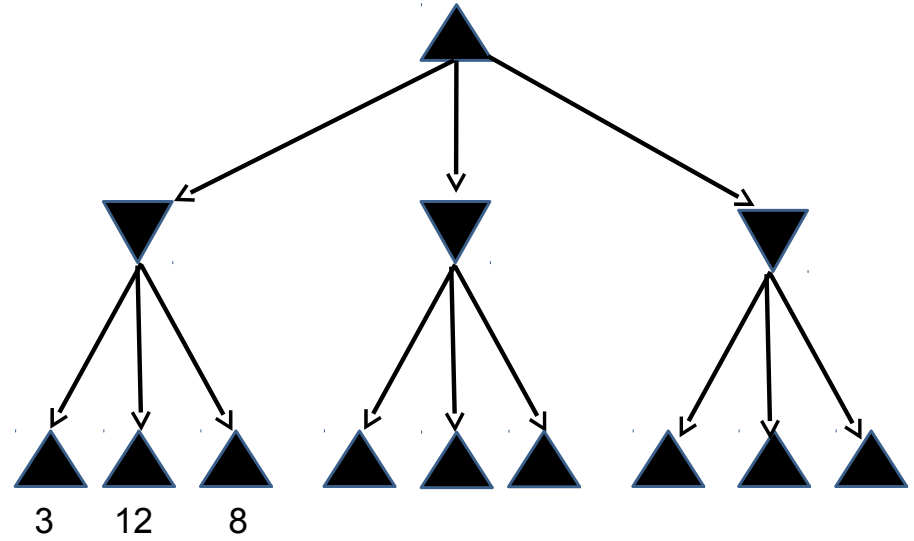
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Alpha-Beta Pruning



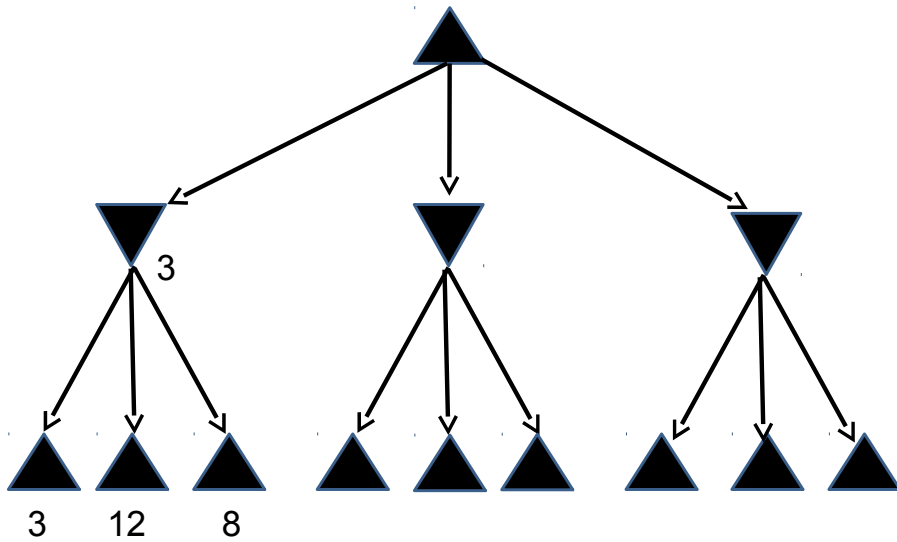
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Alpha-Beta Pruning



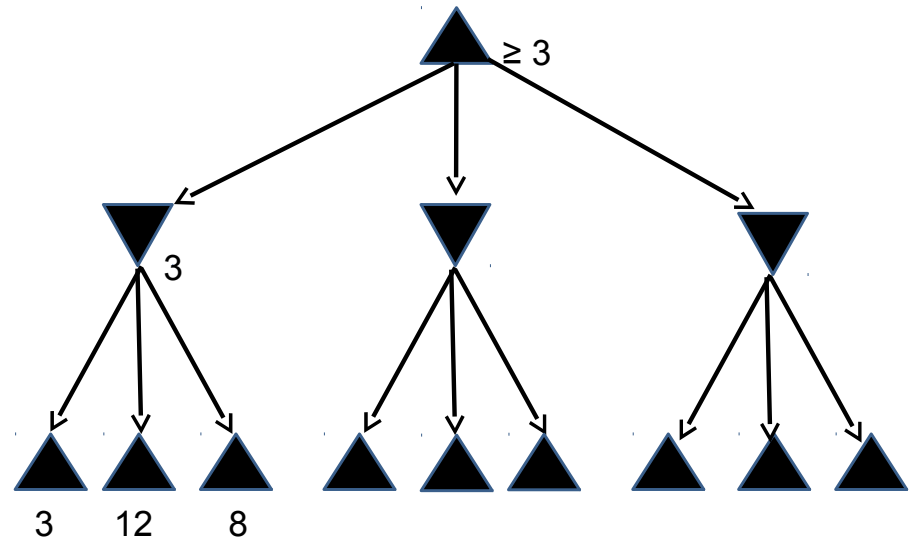
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Alpha-Beta Pruning



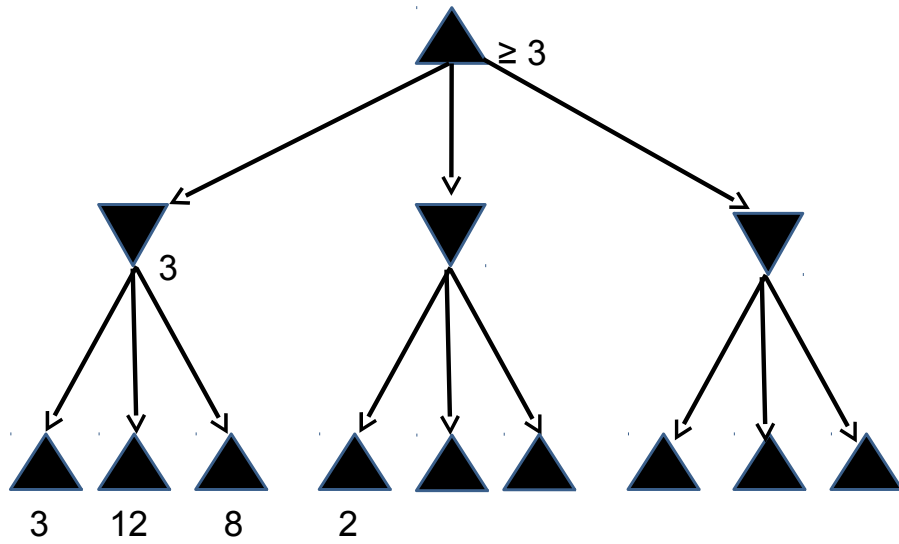
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Alpha-Beta Pruning



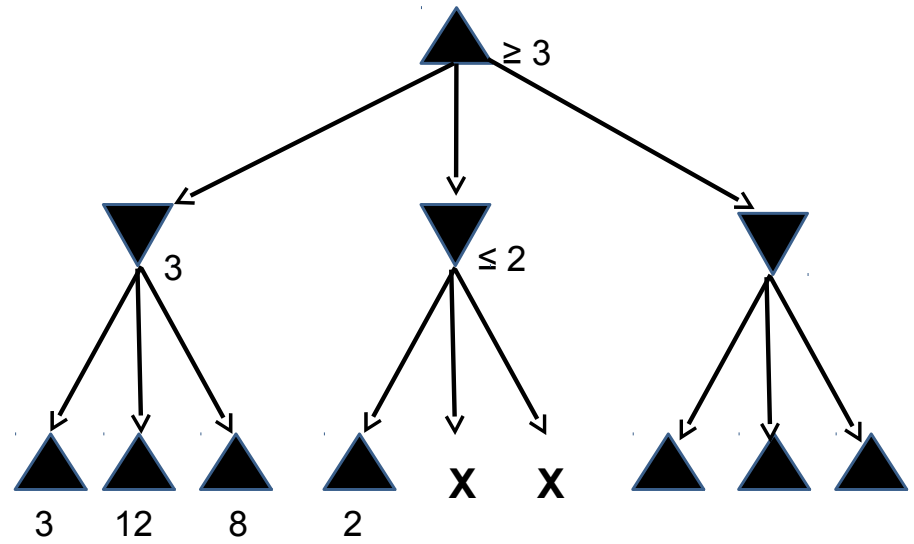
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Alpha-Beta Pruning



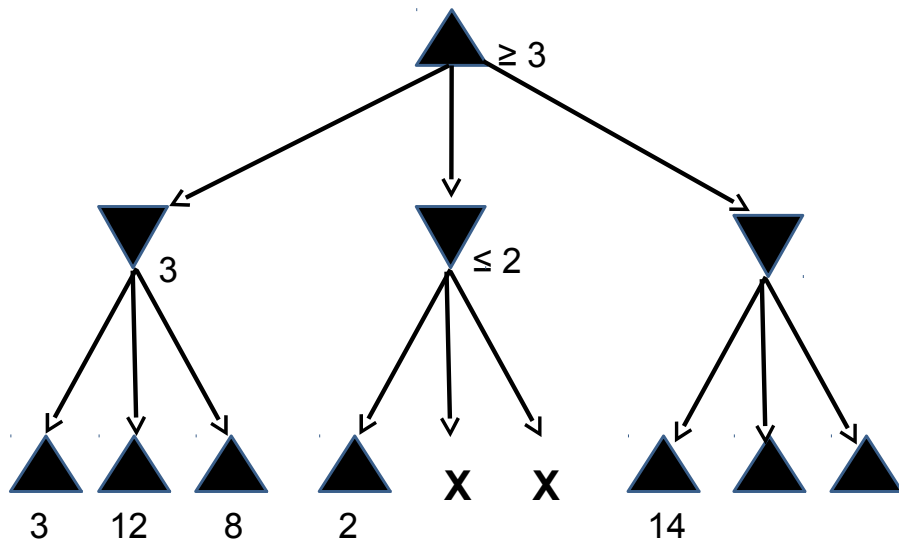
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Alpha-Beta Pruning



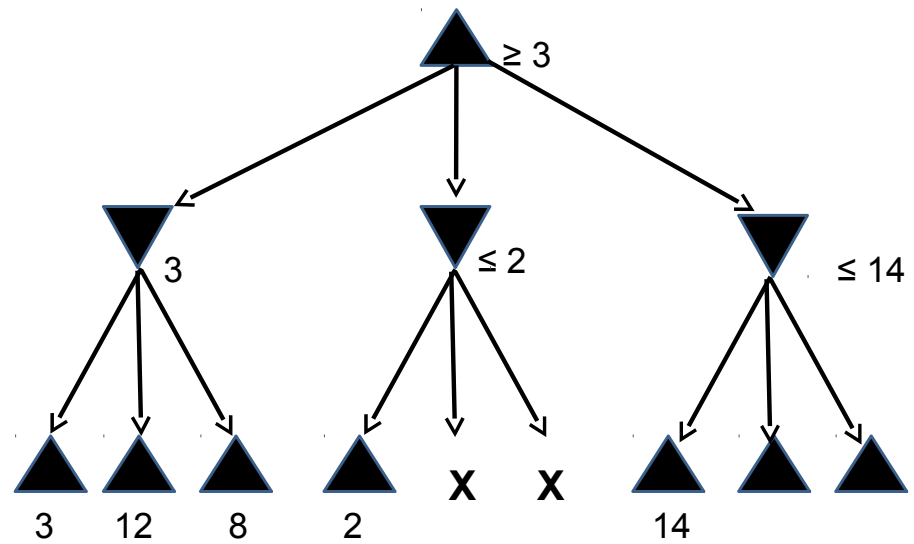
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Alpha-Beta Pruning



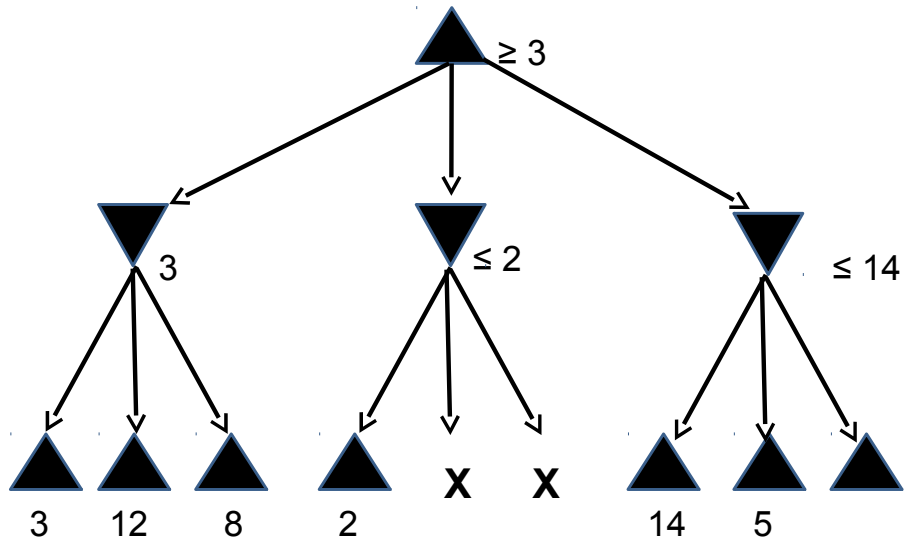
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Alpha-Beta Pruning



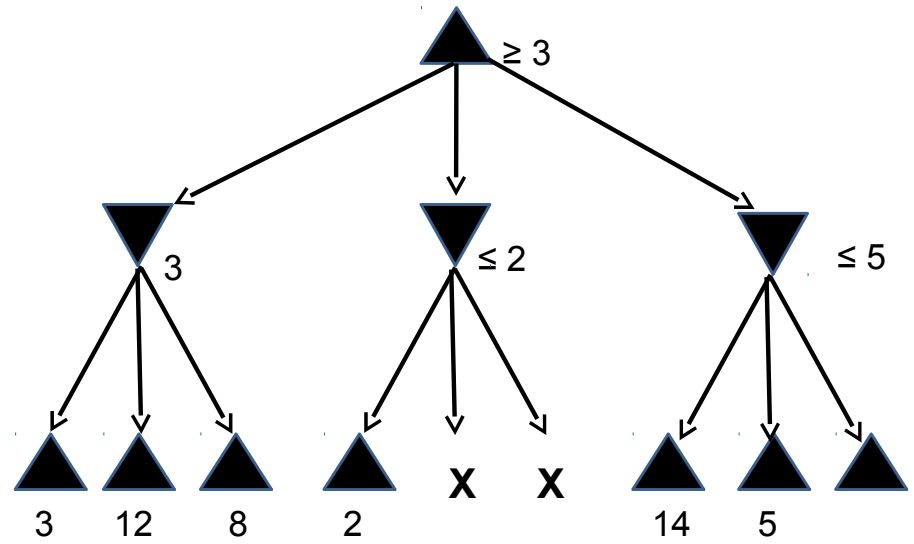
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Alpha-Beta Pruning



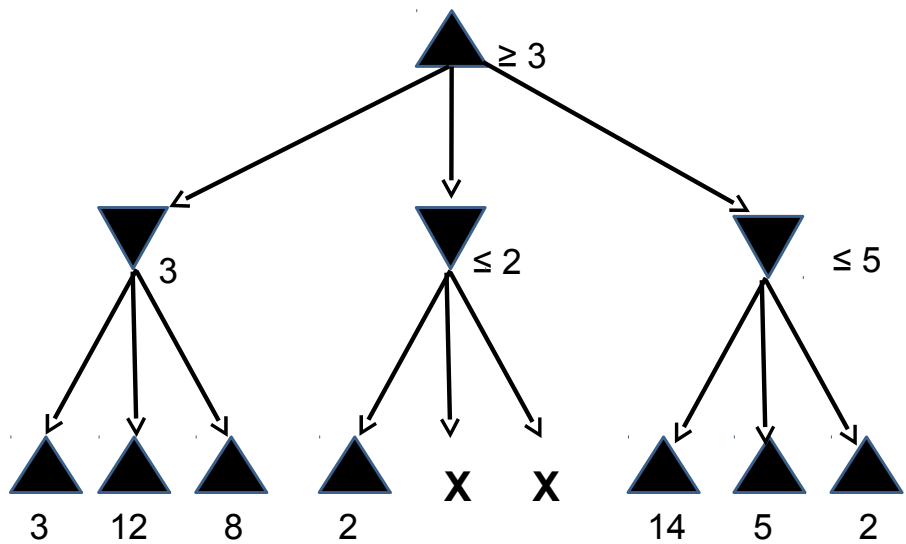
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Alpha-Beta Pruning



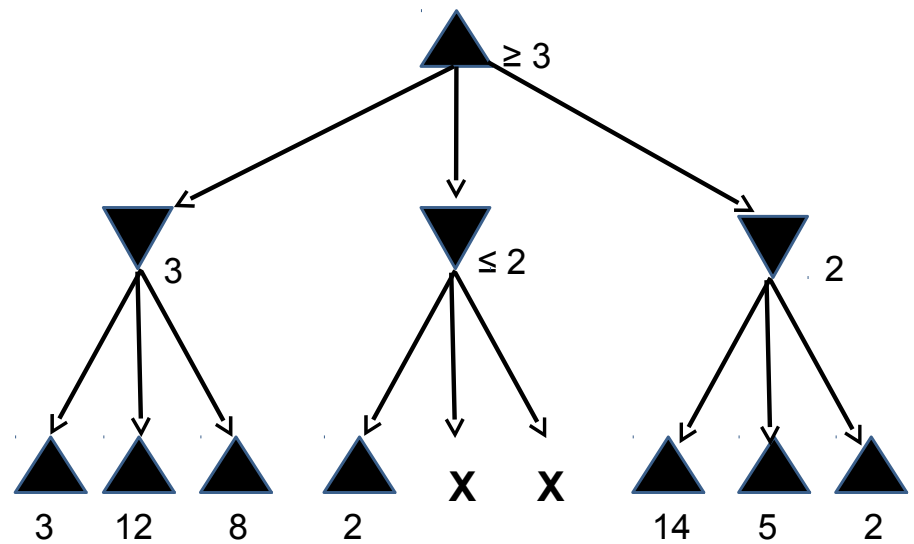
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Alpha-Beta Pruning



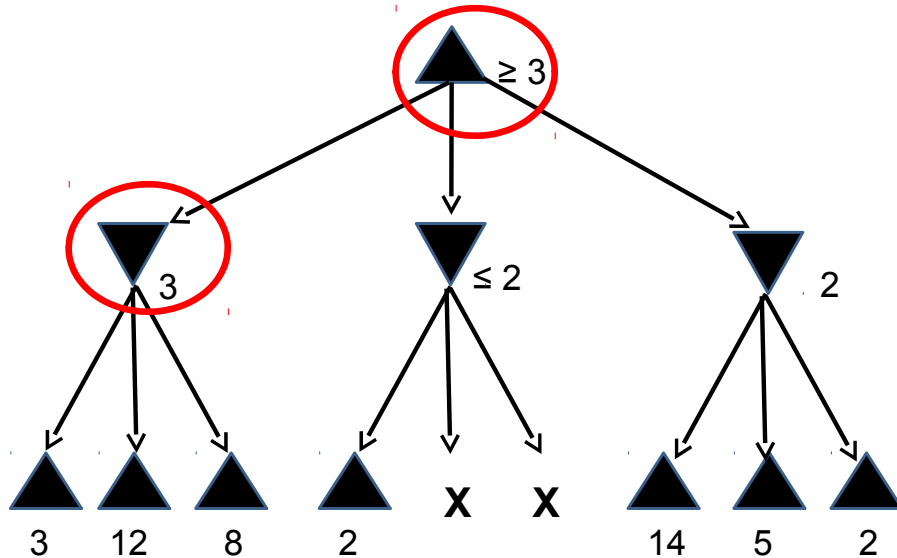
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Alpha-Beta Pruning



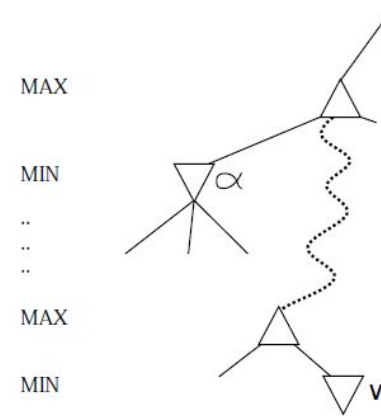
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Alpha-Beta Pruning



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Why is it called alpha-beta?



α is the best value (to MAX) found so far off the current path
 If V is worse than α , MAX will avoid it \Rightarrow prune that branch
 Define β similarly for MIN

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The Alpha-Beta Algorithm

- alpha (α) is value of best (highest value) choice for MAX
- beta (β) is value of best (lowest value) choice for MIN
- If at a MIN node and value $\leq \alpha$, stop looking, because MAX node will ignore this choice
- If at a MAX node and value $\geq \beta$ beta, stop looking because MIN node will ignore this choice

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Properties of Alpha-Beta

- Pruning does not affect final result
- Good move ordering improves effectiveness of pruning
- With “perfect ordering” time complexity $b^{m/2}$ and so doubles solvable depth
- A simple example of the value of reasoning about which computations are relevant (a form of *meta-reasoning*)
- Unfortunately, 35^{50} is still impossible, so chess not completely soluble

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Cutoffs and Heuristics

- Cut off search according to some cutoff test
 - Simplest is a depth limit
- Problem: payoffs are defined only at terminal states
- Solution: Evaluate the pre-terminal leaf states using *heuristic evaluation function* rather than using the actual payoff function



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Example: Chess (I)

- Assume MAX is white
- Assume each piece has the following material value:
 - pawn = 1
 - knight = 3
 - bishop = 3
 - rook = 5
 - queen = 9
- let w = sum of the value of white pieces
- let b = sum of the value of black pieces

$$\text{Evaluation}(n) = \frac{w - b}{w + b}$$

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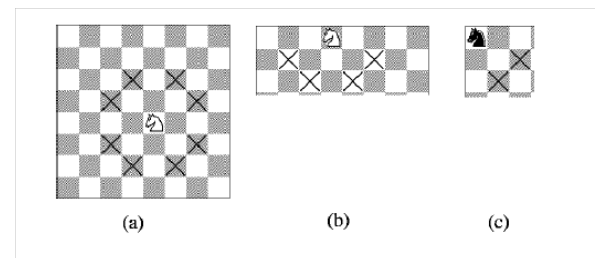
Cutoff Value



- To handle the cutoff, in minimax or alpha-beta search we can make an alteration by making use of a cutoff value
- *MinimaxCutoff* is identical to *MinimaxValue* except
 1. *Terminal* test is replaced by *Cutoff* test, which indicates when to apply the evaluation function
 2. *Utility* is replaced by *Evaluation* function, which estimates the position's utility

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Example: Chess (II)



- The previous evaluation function naively gave the same weight to a piece regardless of its position on the board...
 - Let X_i be the number of squares the i -th piece attacks
 - $\text{Evaluation}(n) = \text{piece}_1.\text{value} * X_1 + \text{piece}_2.\text{value} * X_2 + \dots$

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Example: Chess (III)

- Heuristics based on database search
 - Statistics of wins in the position under consideration
 - Database defining perfect play for all positions involving X or fewer pieces on the board (endgames)
 - Openings are extensively analysed, so can play the first few moves “from the book”

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Deterministic Games in Practice

- Draughts: Chinook ended 40-year-reign of human world champion Marion Tinsley in 1994. Used an endgame database defining perfect play for all positions involving 8 or fewer pieces on the board, a total of 443,748,401,247 positions
- Chess: Deep Blue defeated human world champion Gary Kasparov in a six-game match in 1997. Deep Blue searches 200 million positions per second, used very sophisticated evaluation, and undisclosed methods for extending some lines of search up to 40 ply

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Deterministic Games in Practice

- Othello: human champions refuse to compete against computers, who are too good
- Go: a challenging game for AI ($b > 300$) so progress much slower with computers. AlphaGo was a recent breakthrough



See more at: [University of Alberta GAMES Group](#)

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Summary

- Games have been an AI topic since the beginning. They illustrate several important features of AI:
 - perfection is unattainable so must approximate
 - good idea to think about what to think about
 - uncertainty constrains the assignment of values to states
 - optimal decisions depend on information state, not real state
- Next lecture:
 - We have now finished with the topic of [search](#) so we will move on to [knowledge representation](#)

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